rum magnitudinum. Neque enim ſpectatur in hoc lemmate magnitudo momentorum, ſed prima naſcentium proportio. Eodem recidit ſi loco momentorum uſurpentur vel velocitates incrementorum ac decrementorum (quas etiam motus, mutationes & fluxiones quantitatum nominare licet) vel finitæ quævis quantitates velocitatibus hiſce proportionales. Lateris autem cujuſque generantis coefficiens eſt quantitas, quæ oritur applicando genitam ad hoc latus.

Igitur ſenſus lemmatis eſt, ut, ſi quantitatum quarumcunque perpetuo motu creſcentium vel decreſcentium A, B, C, &c. momenta, vel his proportionales mutationum velocitates dicantur a, b, c, &c. momentum vel mutatio geniti rectanguli AB fuerit $aB+bA$, & geniti contenti ABC momentum fuerit $aBC+bAC+cAB$: & genitarum dignitatum A^2, A^3, A^4, $A^{\frac{1}{2}}$, $A^{\frac{3}{2}}$, $A^{\frac{1}{3}}$, $A^{\frac{2}{3}}$, A^{-1}, A^{-2}, & $A^{-\frac{1}{2}}$ momenta $2aA$, $3aA^2$, $4aA^3$, $\frac{1}{2}aA^{-\frac{1}{2}}$, $\frac{3}{2}aA^{\frac{1}{2}}$, $\frac{1}{3}aA^{-\frac{2}{3}}$, $\frac{2}{3}aA^{-\frac{1}{3}}$, $-aA^{-2}$, $-2aA^{-3}$, & $-\frac{1}{2}aA^{-\frac{3}{2}}$ reſpective. Et generaliter, ut dignitatis cujuſcunque $A^{\frac{n}{m}}$ momentum fuerit $\frac{n}{m}aA^{\frac{n-m}{m}}$. Item ut genitæ A^2B momentum fuerit $2aAB+bA^2$; & genitæ $A^3B^4C^2$ momentum $3aA^2B^4C^2+4bA^3B^3C^2+2cA^3B^4C$; & genitæ $\frac{A^3}{B^2}$ ſive A^3B^{-2} momentum $3aA^2B^{-2}-2bA^3B^{-3}$: & ſic in cæteris. Demonſtratur vero lemma in hunc modum.

Caſ. 1. Rectangulum quodvis motu perpetuo auctum AB, ubi de lateribus A & B deerant momentorum dimidia $\frac{1}{2}a$ & $\frac{1}{2}b$, fuit $A-\frac{1}{2}a$ in $B-\frac{1}{2}b$, ſeu $AB-\frac{1}{2}aB-\frac{1}{2}bA+\frac{1}{4}ab$; & quam primum latera A & B alteris momentorum dimidiis aucta ſunt, evadit $A+\frac{1}{2}a$ in $B+\frac{1}{2}b$ ſeu $AB+\frac{1}{2}aB+\frac{1}{2}bA+\frac{1}{4}ab$. De hoc rectangulo ſubducatur rectangulum prius, & manebit exceſſus $aB+bA$. Igitur laterum incrementis totis a & b generatur rectanguli incrementum $aB+bA$. *Q. E. D.*

Caſ. 2. Ponatur AB ſemper æquale G, & contenti ABC ſeu GC momentum (per caſ. 1.) erit $gC+cG$, id eſt (ſi pro G & g ſcribantur AB & $aB+bA$) $aBC+bAC+cAB$. Et par eſt ratio contenti ſub lateribus quotcunque. *Q. E. D.*

Caſ.

Caſ. 3. Ponantur latera A, B, C ſibi mutuo ſemper æqualia; & ipſius A^2, id eſt rectanguli AB, momentum $aB+bA$ erit $2aA$, ipſius autem A^3, id eſt contenti ABC, momentum $aBC+bAC+cAB$ erit $3aA^2$. Et eodem argumento momentum dignitatis cujuſcunque A^n eſt naA^{n-1}. *Q. E. D.*

Caſ. 4. Unde cum $\frac{1}{A}$ in A ſit 1, momentum ipſius $\frac{1}{A}$ ductum in A, una cum $\frac{1}{A}$ ducto in a erit momentum ipſius 1, id eſt, nihil. Proinde momentum ipſius $\frac{1}{A}$ ſeu ipſius A^{-1} eſt $\frac{-a}{A^2}$. Et generaliter cum $\frac{1}{A^n}$ in A^n ſit 1, momentum ipſius $\frac{1}{A^n}$ ductum in A^n una cum $\frac{1}{A^n}$ in naA^{n-1} erit nihil. Et propterea momentum ipſius $\frac{1}{A^n}$ ſeu A^{-n} erit $-\frac{na}{A^{n+1}}$. *Q. E. D.*

Caſ. 5. Et cum $A^{\frac{1}{2}}$ in $A^{\frac{1}{2}}$ ſit A, momentum ipſius $A^{\frac{1}{2}}$ ductum in $2A^{\frac{1}{2}}$ erit a, per caſ. 3: ideoque momentum ipſius $A^{\frac{1}{2}}$ erit $\frac{a}{2A^{\frac{1}{2}}}$ ſive $\frac{1}{2}aA^{-\frac{1}{2}}$. Et generaliter ſi ponatur $A^{\frac{m}{n}}$ æquale B, erit A^m æquale B^n, ideoque maA^{m-1} æquale nbB^{n-1}, & maA^{-1} æquale nbB^{-1} ſeu $nbA^{-\frac{m}{n}}$, ideoque $\frac{m}{n}aA^{\frac{m-n}{n}}$ æquale b, id eſt, æquale momento ipſius $A^{\frac{m}{n}}$. *Q. E. D.*

Caſ. 6. Igitur genitæ cujuſcunque A^mB^n momentum eſt momentum ipſius A^m ductum in B^n, una cum momento ipſius B^n ducto in A^m, id eſt $maA^{m-1}B^n+nbB^{n-1}A^m$; idque ſive dignitatum indices m & n ſint integri numeri vel fracti, ſive affirmativi vel negativi. Et par eſt ratio contenti ſub pluribus dignitatibus. *Q. E. D.*

Corol. 1. Hinc in continue proportionalibus, ſi terminus unus datur, momenta terminorum reliquorum erunt ut iidem termini multipli-

cati

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